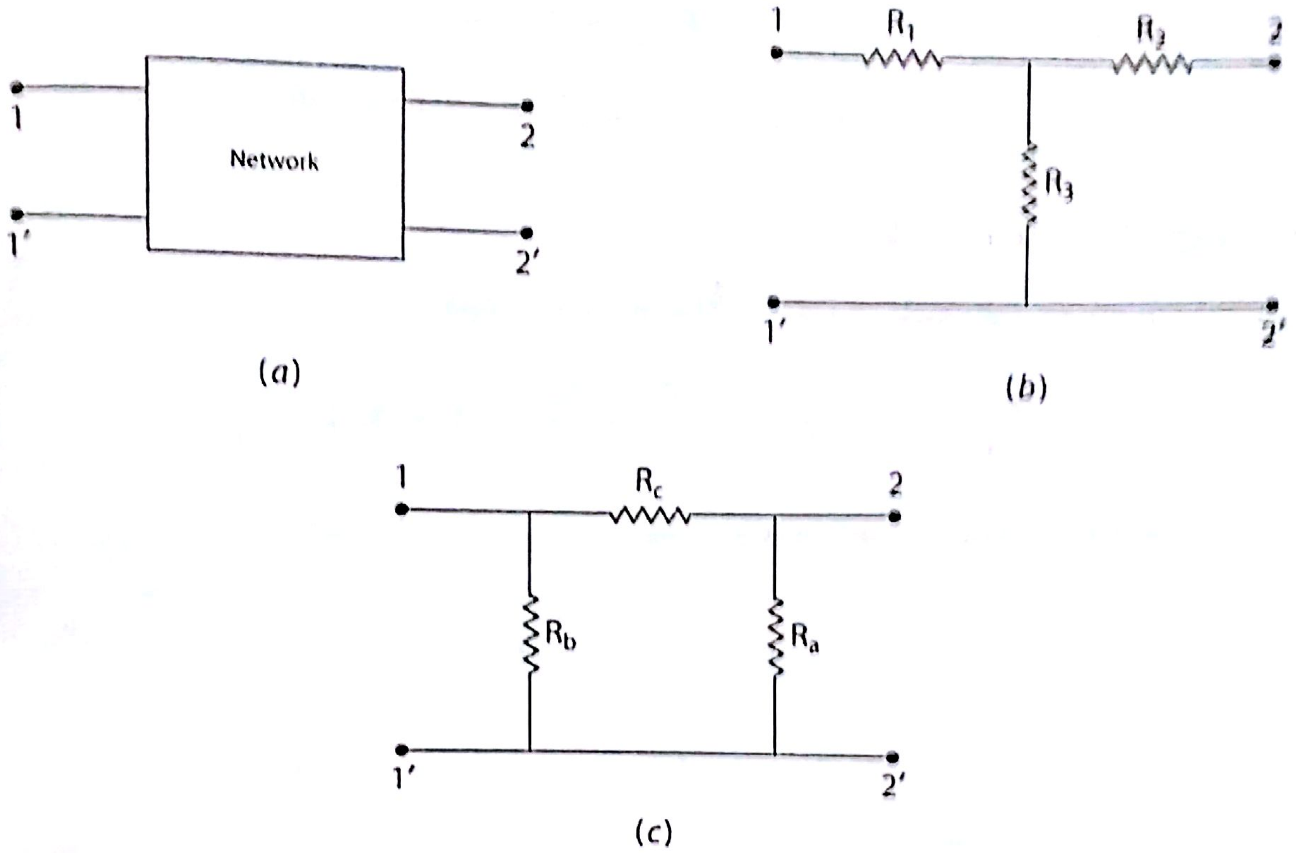
7.10 T- and π-Networks

Two four-terminal or two-port networks are said to be equivalent to each other if they have identical terminal properties, i.e., for the same input they yield identical outputs. Sometimes the analysis of a complex circuit can be simplified by considering a simpler equivalent circuit. Here we show that any two-port linear resistive network can be replaced by an equivalent T-network or a π-network.



(a) (b) (c)

Fig 7.10-1: ***(a)*** Arbitrary network, **(b)** T-equivalent, **(c)** $\overset{\pi}{\text{p}}$-equivalent.

Fig 7.10-1 shows an arbitrary complicated network and its T- and π-equivalent circuits. Now for the circuits to be equivalent the values of terminal voltages and currents must be identical. So the open and short-circuit resistances measured at the terminals must also be identical.

T-equivalent network

Resistance at $1, 1'$ with $2, 2'$ short circuited is

$$R_{1S} = R_1 + \frac{R_2 R_3}{R_2 + R_3}. \tag{7.10-1}$$

Resistance at $1, 1'$ with $2, 2'$ open circuited is

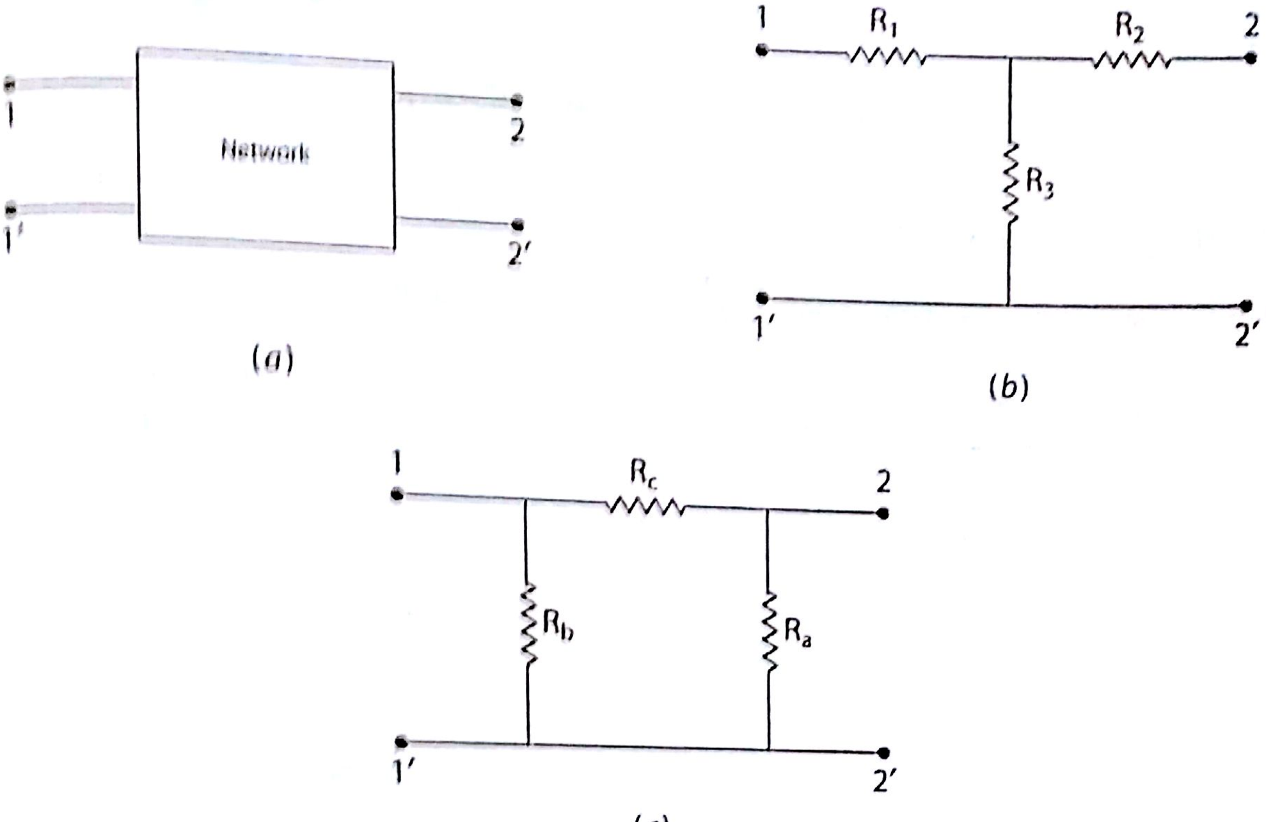
$$R_{1O} = R_1 + R_3. \tag{7.10-2}$$

Resistance at $2, 2'$ with $1, 1'$ open circuited is

$$R_{2O} = R_2 + R_3. \tag{7.10-3}$$

Solving these equations we can find the required values of R_1, R_2 and R_3 in terms of the measurements R_{1S}, R_{1O}, R_{2O} of the original network. Subtracting Eq. (7.10-1) from Eq. (7.10-2) and then using Eq. (7.10-3) we get

$$R_{1O} - R_{1S} = \frac{R_3^2}{R_{2O}} \quad \text{or} \quad R_3 = \sqrt{R_{2O}(R_{1O} - R_{1S})} \tag{7.10-4}$$



(a) (b) (c)

Fig 7.10-1: **(*a*)** Arbitrary network, **(b)** T-equivalent, **(c)** $\overset{\pi}{\text{p}}$-equivalent.

Fig 7.10-1 shows an arbitrary complicated network and its T- and π-equivalent circuits. Now for the circuits to be equivalent the values of terminal voltages and currents must be identical. So the open and short-circuit resistances measured at the terminals must also be identical.

T-equivalent network

Resistance at 1, 1′ with 2, 2′ short circuited is

$$R_{1S} = R_1 + \frac{R_2 R_3}{R_2 + R_3}. \qquad (7.10\text{-}1)$$

Resistance at 1, 1′ with 2, 2′ open circuited is

$$R_{1O} = R_1 + R_3. \qquad (7.10\text{-}2)$$

Resistance at 2, 2′ with 1, 1′ open circuited is

$$R_{2O} = R_2 + R_3. \qquad (7.10\text{-}3)$$

Solving these equations we can find the required values of R_1, R_2 and R_3 in terms of the measurements R_{1S}, R_{1O}, R_{2O} of the original network. Subtracting Eq. (7.10-1) from Eq. (7.10-2) and then using Eq. (7.10-3) we get

$$R_{1O} - R_{1S} = \frac{R_3^2}{R_{2O}} \quad \text{or} \quad R_3 = \sqrt{R_{2O}(R_{1O} - R_{1S})}. \qquad (7.10\text{-}4)$$

Now using Eqs. (7.10-2) and (7.10-3) we can find R_1 and R_2 as

$$R_1 = R_{1O} - R_3 \quad \text{and} \quad R_2 = R_{2O} - R_3. \tag{7.10-5}$$

π-equivalent network

For π-equivalent network of Fig 7.10-1(c) we have similarly,

$$R_{1S} = \frac{R_b R_c}{R_b + R_c}, \quad R_{1O} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c}, \quad R_{2O} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \tag{7.10-6}$$

To solve these equations we note that

$$R_{1S} R_{2O} = \frac{R_a R_b R_c}{R_a + R_b + R_c}. \tag{7.10-7}$$

$$\text{and} \quad \sqrt{R_{2O}(R_{1O} - R_{1S})} = \frac{R_a R_b}{R_a + R_b + R_c}. \tag{7.10-8}$$

Thus

$$\begin{aligned} R_c &= \frac{R_{1S} R_{2O}}{\sqrt{R_{2O}(R_{1O} - R_{1S})}}, \\ R_b &= \frac{R_{1S} R_{2O}}{R_{2O} - \sqrt{R_{2O}(R_{1O} - R_{1S})}}, \\ R_a &= \frac{R_{1S} R_{2O}}{R_{1O} - \sqrt{R_{2O}(R_{1O} - R_{1S})}}. \end{aligned} \tag{7.10-9}$$